

Growth Sensitivity of the Nigerian Population and a Prediction for the Future.

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ABSTRACT

From the Google map online, Nigeria is a country located in West Africa with surrounding countries like Benin, Togo and Ghana on its west surrounding; Chad, Burkina Faso and Cameroon on its North East surrounding while Niger on the north surrounding. Nigeria comprises of 36 states and its federal capital territory called Abuja with its largest city called Lagos. Nigeria has a total land mass of 923,768km², i.e. 356669 square meters with its water percentage estimate at 1.4. It was recognized on the 1st of October 1960 and the later declared a republic on the 1st of October 1963. We will be focusing on the predicted and sensitivity growth model which is applied to model the population of Nigeria as from 2015 to 2125 using Thomas R. Malthus population growth model. The carrying capacity, sensitivity and important coefficients governing the growth of the population in Nigeria has been determined. The outcome shows that the carrying capacity for the Nigerian population is 808719320.62, while the coefficient x and y are 0.03 and 3.71×10^{-11} . According to this model, Nigeria's population growth rate is 3% per annum. The population prediction for Nigeria by 2125 will be 802,430,000 while the population sensitivity will be 189,900.

KEYWORDS: *Predicted growth model; Carrying capacity; Vital coefficients; Population Sensitivity; Population growth rate.*

INTRODUCTION:

Population projection of Nigeria is important in planning and making sensitive and critical decision economically, politically, socially and also on the demographic development of the nation. Projection population is used for planning for food and water use, health, education, politics and public service. Nigeria is regarded the giant of Africa with a population well over one hundred and eighty six million people making it the most populated nation on the African continent and the seventh most populated nation in the world with over two hundred and fifty ethnic group with different cultures in Nigeria

A nations policy, culture, education, politics, economy, exploration, of natural resources and social activities are influenced by the size and growth of the population of a country. In essence the

planning of a nation by the government and other sectors is dependent on population, demands, natural resources and others. Population growth has a major effect on social, political, economic and environmental development. Hence mathematical model uses the technique of mathematics and computation to predict population growth in logistic growth model which establishes many fields of modeling and forecasting. Some factors such as death rates can have a reduced effect on the population growth. But birth control rate will be the best way of reducing the population growth of Nigeria. Y Yan and EON Ekaka-a (2011) studied the mathematical model of population system. Augustus et. al. (2011) studied about the mathematical modeling of the Rwanda's population growth while Ofori et. al. (2013) did a study on the mathematical model of the Ghanaian population. Therefore in this paper we will be determining the carrying capacity, sensitivity and the vital coefficients governing population growth.

2. METHODOLOGY:

In a research, we attain a process of reaching solutions that are dependent on problems through a collection that is systematic with analysis and interpretation of data. In this paper data were collected National Bureau of Statistics of Nigeria. The Logistic growth mathematical model was used to compute predicted population values employing MATLAB.

3 DEVELOPMENT OF THE MODEL

Thomas R. Malthus In 1798 developed a population growth model. Let $G(t)$ represent the population of specie at time t and x represent the difference between its birth rate and death rate. If this population is isolated, then $\frac{dG}{dt}$, the sensitivity of the population (rate of change of the population) which equals $xG(t)$ where x is a constant that does not change with either time or population. The differential equation governing population growth is

$$\frac{dG}{dt} = xG(t) \tag{1}$$

Where t represent the time period and x is the Malthusian growth factor. The equation is a non-homogeneous linear first order differential equation known as Malthusian law of population growth. $G(t)$ takes on only integral values and it is a discontinuous function of t .

$$G(t) = G_0 e^{xt} \tag{2}$$

Species satisfying the Malthusian law of population growth grows exponentially with time. A mathematical model with carrying capacity is considered since increased population size will cause a decrease in food availability, increased waste product, increase in death rate, increase in poverty due to poor economic planning and policy, and others that are not mentioned.

Verhulst (1838), studied the dependence of population growth on its size and its carrying capacity (maximum supportable population). He modified Malthus's Model to make the population size proportional to both the previous population and a new term

$$\frac{x-yG(t)}{x} \tag{3}$$

Where x and y are the population coefficient. This term reflects how far the population is from its maximum limit. However, as the population grows to $\frac{x}{y}$, $\frac{x-yG(t)}{x} \rightarrow 0$, providing The modified equation using the new term is

$$\frac{dG}{dt} = \frac{xG(t)[x-yG(t)]}{x} \tag{4}$$

The above equation is called the logistic law of population growth. Putting $G = G_0$ for $t = 0$, where G_0 represents the population at some specified time, $t = 0$, equation (4) becomes

$$\frac{d}{dt} G(t) = \frac{xG-yG^2}{x} \tag{5}$$

Separating the variables in equation (5) and integrating, we obtain

$$\int \frac{1}{x} \left(\frac{1}{G} + \frac{y}{x-yG} \right) dG = t + c$$

so that

$$\frac{1}{x} (\log G - \log(x - yG)) = t + c \tag{6}$$

For $t = 0, G = G_0$ we observe that $\frac{1}{x} (\log G_0 - \log(x - G_0))$ becomes

$$\frac{1}{x} (\log G - \log(x - yG)) = t + \frac{1}{x} (\log G_0 - \log(x - yG_0)) \tag{7}$$

$$G = \frac{\frac{x}{y}}{1 + \left(\frac{\frac{x}{y}}{G_0} - 1 \right) e^{-xt}} \tag{8}$$

Take the limit of equation (8) as $t \rightarrow \infty$ for $x > 0$

$$G_{max} = \lim_{t \rightarrow \infty} G = \frac{x}{y} \tag{9}$$

Suppose $t = 1, t = 2$ with values of G, G_1 and G_2

$$\frac{y}{x} (1 - e^{-x}) = \frac{1}{G_1} - \frac{e^{-x}}{G_0}, \quad \frac{y}{x} (1 - e^{-2x}) = \frac{1}{G_2} - \frac{e^{-2x}}{G_0} \tag{10}$$

Divide both equations to remove $\frac{y}{x}$

$$1 + e^{-x} = \frac{\frac{1}{G_2} - \frac{e^{-2x}}{G_0}}{\frac{1}{G_1} - \frac{e^{-x}}{G_0}} \tag{11}$$

Put e^{-x} in first equation in equation (10)

$$\frac{y}{x} = \frac{G_1^2 - G_0 G_2}{G_1 (G_0 G_1 - 2G_0 G_2 + G_1 G_2)} \tag{12}$$

$$G_{max} = \lim_{t \rightarrow \infty} G = \frac{x}{y} = \frac{G_1(G_0G_1 - 2G_0G_2 + G_1G_2)}{G_1^2 - G_0G_2} \quad (13)$$

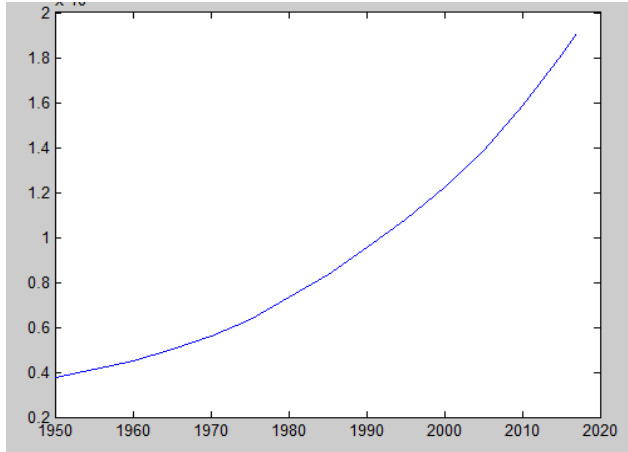
4 Results

The table 1 below shows the population of Nigeria from 1950 to 2019(inclusive). Another table 2 is created below to show the population prediction from 2015 to 2119. From table 2 below, we assume $t = 0, 1, 2$ corresponding to the years 2015, 2016 and 2017 respectively. The population value for G_0, G_1, G_2 are 181181744, 185989640 and 190886311 respectively.

Table 1: NIGERIAN POPULATION DATA

S/N	G	YEAR	ACTUAL POLPULATION
1.		1950	37,859744
2.		1955	41085563
3.		1960	45137812
4.		1965	50127214
5.		1970	55981400
6.		1975	63373572
7.		1980	73460724
8.		1985	83613300
9.		1990	95269988
10.		1995	108011465
11.		2000	122352009
12.		2005	138939478
13.		2010	158578261
14.	G_0	2015	181181744
15.	G_1	2016	185989640
16.	G_2	2017	190886311

Figure 1: Plot of actual population from 1950 to 2019



Substituting value for G_0 , G_1 , G_2 into equation (13) we will have $G_{max} = \frac{x}{y} = 808719321.62$.

This value will be the predicted carrying capacity of the Nigerian population. In equation (11) we got $e^{-x} \approx 0.97$ so that $x \approx -\log_e 0.97$ and the value of

$$x \approx 0.03 \tag{14}$$

This means that the growth of the Nigerian population per year is 3%

Since $\frac{x}{y} = 808719321.62$ and $x = 0.03$ therefore, $y = 3.71 \times 10^{-11}$

Substitute G_0 , $\frac{x}{y}$ and e^{-x} into equation (8)

$$G = \frac{808719320.62}{1+(3.4636 \times 0.97^t)} \tag{15}$$

The growth sensitivity for the predicted population becomes

$$\frac{dG}{dt} = \frac{-808719320.62 \times 3.4636 \times 0.97^t \times \log_e 0.97}{[1+(3.4636 \times 0.97^t)]^2} \tag{16}$$

T	Year	G(1×10^8) Predicted Growth	(Growth Sensitivity 1×10^6)
0	2015	1.8118	4.2823
5	2020	2.0349	4.6385
10	2025	2.2754	4.9807
15	2030	2.5325	5.2983
20	2035	2.8047	5.5801
25	2040	3.0898	5.8156
30	2045	3.3853	5.9950
35	2050	3.6882	6.1107
40	2055	3.9952	6.1574
45	2060	4.3028	6.1329
50	2065	4.6073	6.0385
55	2070	4.9055	5.8784
60	2075	5.1942	5.6596

65	2080	5.4706	5.3912
70	2085	5.7327	5.0837
75	2090	5.9785	4.7481
80	2095	6.2072	4.3952
85	2100	6.4179	4.0349
90	2105	6.6107	3.6762
95	2110	6.7857	3.3262
100	2115	6.9436	2.9908
105	2120	7.0851	2.6741
110	2125	7.2113	2.3789
115	2130	7.3234	2.1068
120	2135	7.4224	1.8584
125	2140	7.5096	1.6336
130	2145	7.5862	1.4316
135	2150	7.6531	1.2512
140	2155	7.7116	1.0909
145	2160	7.7625	0.9492
150	2165	7.8068	0.8244
155	2170	7.8452	0.7150
160	2175	7.8785	0.6192
165	2180	7.9073	0.5356
170	2185	7.9323	0.4628
175	2190	7.9538	0.3996
180	2195	7.9724	0.3448
185	2200	7.9884	0.2973
190	2205	8.0022	0.2562
195	2210	8.0141	0.2206
200	2215	8.0243	0.1899

Figure 2: Plot for predicted growth from 2015 to 2210

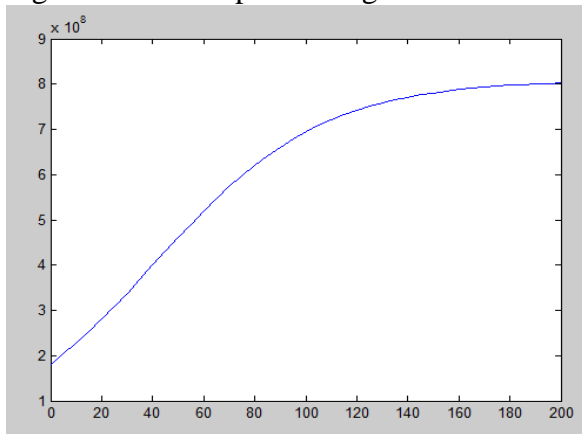
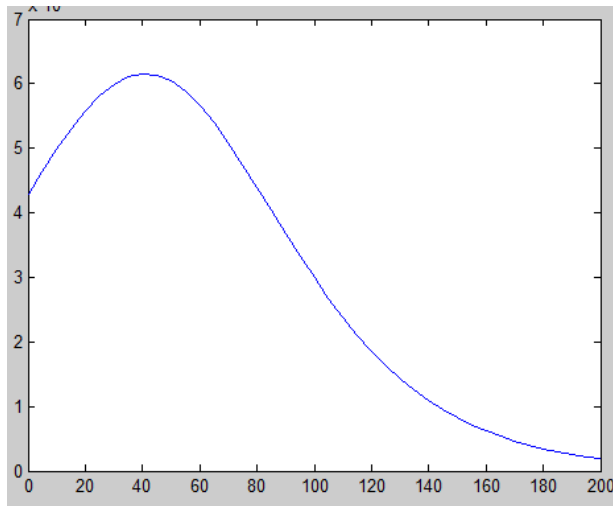


Figure 3: Plot for Growth sensitivity from 2015 to 2210



Conclusion (Observation and projection for the future)

Table 1 shows that the Nigerian population from 1950 to 2017 increases from 3.7860×10^8 million to 1.9089×10^8 million. In figure 1 it is observed that the population over six decades from 1950 till 2017, the population growth increases.

In figure 2 the growth increases exponentially as the year increases in the first century from 2015 to 2115 with population increase from 1.8118×10^8 million to 6.9436×10^8 million. It is observed that the population for the next century from 2115 to 2215 increases slightly and then tends to stability. Hence the population growth becomes slightly stable and the stable beyond 2215.

In figure 3 the growth sensitivity increases for the next three and half decades from 2015 to 2040. This means that the growth rate is still under some form of control or bound. After 2040 the growth sensitivity tends to zero. At this year the growth rate is out of control showing a decline in growth sensitivity which implies that everything should be done by the government both in policy to ensure that the population growth is been checked early enough and controlled. This will bring the stability of the population growth to an early time of the first century and not a later time of the second century. Hence the growth sensitivity shows that stability of the population growth is tending to zero.

Finally, at zero the growth sensitivity is stable implying that in the second century the population growth of Nigeria beyond 2215 will attain stability.

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